Value Based Decision Control: Management and Decision Support and/or Control of Complex Systems

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IICT-BAS

Секция "Комуникационни услуги и системи"

Ценностно базирани управленски решения: мениджмънт и поддържане на решенията и/или управление на сложни системи

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Ценностно базираните решения
Value-driven decision

- Ценностно базираните решения е среда в която вземащия решение базира своята стратегия на оптимални решения на основата на целеви функции и модели, в които се включват основните важни характеристики и атрибути на изследваната система.

- **Value-driven design** (**VDD**) is a systems engineering strategy based on microeconomics which enables multidisciplinary design optimization. Value-driven design is being developed by the American Institute of Aeronautics and Astronautics, through a program committee of government, industry and academic representatives.[1] In parallel, the US Defense Advanced Research Projects Agency has promulgated an identical strategy, calling it **Value centric design**, on the F6 Program. At this point, the terms *value-driven design* and *value centric design* are interchangeable. The essence of these strategies is that design choices are made to maximize system value rather than to meet performance requirements.

- https://en.wikipedia.org/wiki/Value-driven_design

Въвеждащ пример: управление на портфолио (portfolio allocation)

- **The principles of rationality and market efficiency lead to modern portfolio theory, and to the Black–Scholes theory for option valuation. Financial economic formally considers investment under certainty and uncertainty (risk) and hence contributes to determine rational (even optimal) financial business strategy. Consider a non-risky asset S^0 and risky one S. The financial market is model by the Black-Scholes stochastic differential equation given by:**

$$dS_t^0 = S_t^0 \, r dt \quad \text{and} \quad dS_t = S_t \mu dt + \sigma dW_t \, .$$

OPTIMAL INVESTMENT POLICY: BLACK–SCHOLES STOCHASTIC MODEL

Following the scientific sources the Black-Scholes model presents a stochastic differential equation given by:

$$dX_t^{\pi} = \pi_t X_t^{\pi} \frac{dS_t}{S_t} dt + (X_t^{\pi} - \pi_t X_t^{\pi}) \frac{dS_t^0}{S_t^0} =$$
$$= (rX_t^{\pi} + (\mu - r)\pi_t X_t^{\pi}) + \sigma \pi_t X_t^{\pi} dW_t$$

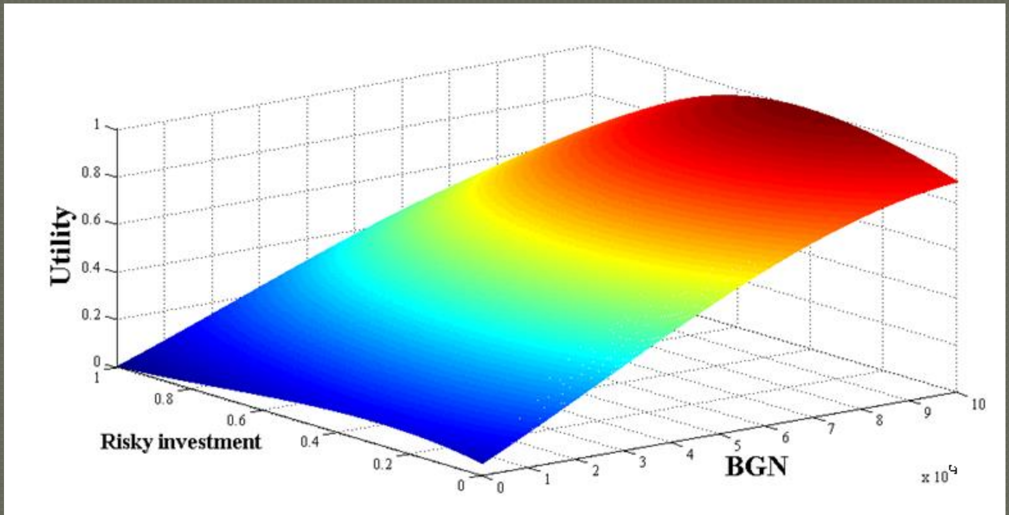
Here r, μ and σ are constants (r = 0.03, μ = 0.05 and σ = 0.3) and W is a one dimensional Brownian motion.

By X_t we denote the state space vector of the controlled time continuous financial market.

The investment policy is defined by a progressively adapted process $\pi=\{\pi_t, t\in[0,T]\}$ where π_t defines the amount $(X_t\ \pi_t)$ $(\pi_t \in[0,1])$ invested in the risky process at time t. The remaining wealth $(X_t - \pi_t\ X_t)$ at the same moment t is invested in the non-risky process. The time period T is 50 weeks.

OPTIMAL INVESTMENT POLICY: HUMAN PREFERENCES AND UTILITY DESCRIPTION

We assume that the outcome set X is a two-attribute product set V×W, with generic element x= (v, w). The sets V and W are attribute sets where V designates the first attribute- the amount π_t, ($X_t\ \pi_t$, $\pi_t \in [0,1]$) invested in the risky process and W designates the second attribute, the quantity of money in BGN's.



The objective of the investor (decision maker-DM) is to choose the control (the amount π_t invested in the risky process) so as to maximize the expected utility of his terminal wealth at moment T, i.e:

$$V(t,x) := \sup_{\pi \in [0,1]} \mathrm{E}[U(X_T^{t,x,\pi})],$$

Where $X_T^{t,x,\pi}$ is the solution of the controlled stochastic differential equation with initial condition (initial wealth) x at time t . For the liquidation value it is supposed that if the state space vector is zero in a moment t then it remains zero until the end T .

Continuous-time financial market with control

- **Optimal portfolio allocation –The Black-Scholes model: Consider a financial market consisting of a non-risky asset S_0 and a risky one S_t. The dynamics of the price processes are given by:**

$$dS_t^0 \; = \; S_t^0 r dt \quad \text{and} \quad dS_t \; = \; S_t[\mu dt + \sigma dW_t] \; .$$

- **Here, r=0.03; μ=0.05 and σ=0.3 are some given positive constants and W is a one-dimensional Brownian motion. The investment policy is defined by an F-adapted process $\pi = \{ \pi_t; t \in [0; T]$, where *T is 50 weeks and* where π_t represents the amount invested in the risky asset at time t; The remaining wealth (X_t - π_t;) is invested in the risky asset. Therefore, the liquidation value of a self-financing strategy satisfies the following equation:**

$$\begin{aligned} dX_t^\pi &= \pi_t \frac{dS_t}{S_t} + (X_t^\pi - \pi_t)\frac{dS_t^0}{S_t^0} \\ &= (rX_t + (\mu - r)\pi_t)\, dt + \sigma \pi_t dW_t . \end{aligned}$$

- **The parameter γ is called the relative risk aversion coefficient. The objective of the investor is to choose an allocation of his wealth so as to maximize the expected utility of his terminal wealth:**

$V(t,x)=\sup_\pi E(U(X_t^{t;x;\pi}))$, $\pi \in U_0$,

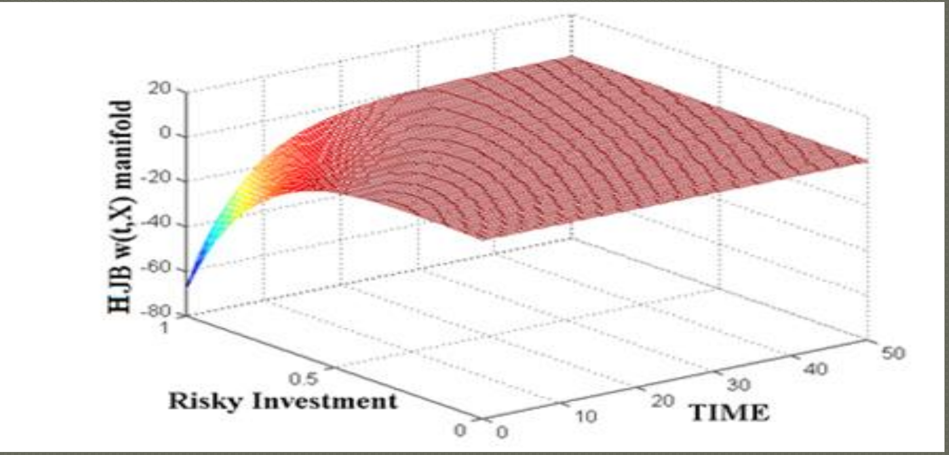
where $X^{t;x;\pi}$ is the solution of stochastic differential equation with initial condition $X_t^{t;x;\pi} = x$.

Continuous-time financial market with control

The optimal control is determined as unique solution of the Hamilton-Jacobi-Bellman (HJB) partial differential equation when the control is a piecewise continuous function, using the dynamical programming principle. The HJB equation has the lollowing presentation where w is the Belman function

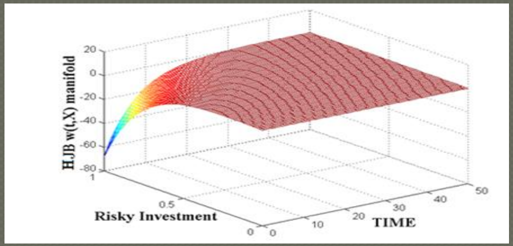
$$\frac{\partial w}{\partial t}(t,x)+\sup_{\pi\in[0,1]}[(rx+(\mu-r)\pi x)\frac{\partial w}{\partial x}(t,x)+\frac{1}{2}\sigma^2\pi^2x^2\frac{\partial^2 w}{\partial^2 x}(t,x)]=0.$$

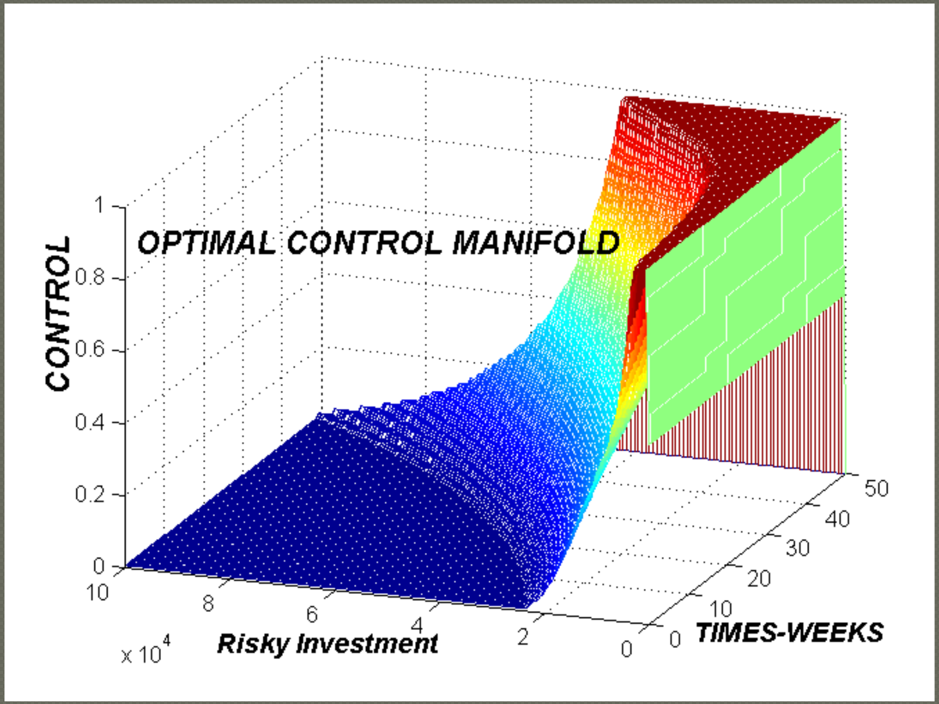
A polynomial approximation of the HJB function w(t, X) is determined ffollowing the presentations in the book of Gabasov and Kirilova:



Continuous-time financial market with control

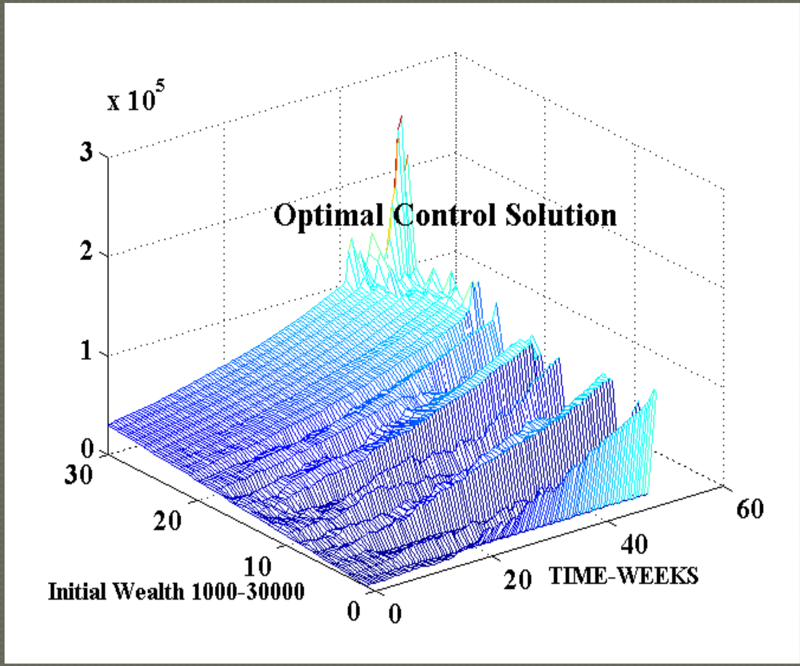
The optimal control manifold is determined based on the Hamilton-Jacobi-Bellman (HJB) function w(t, X):

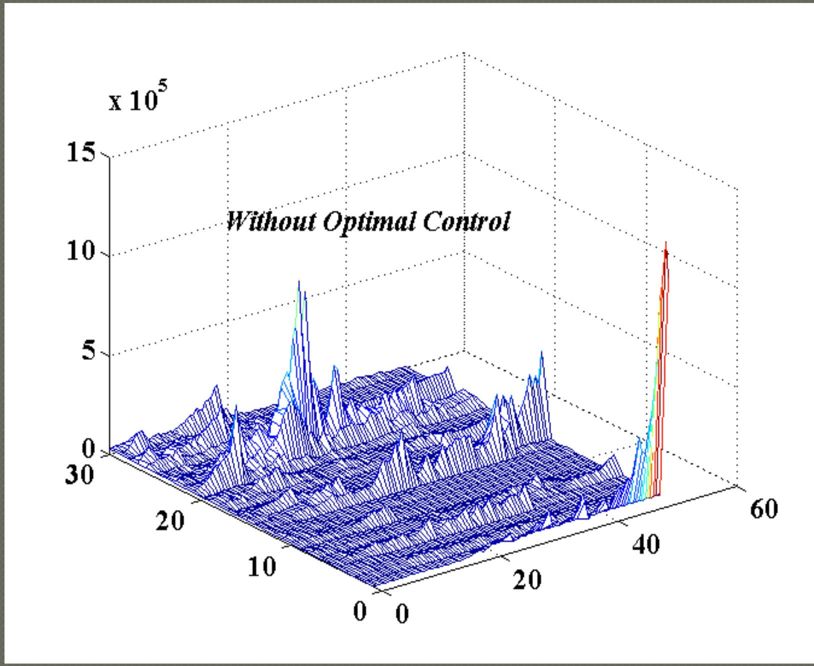




Continuous-time financial market with control

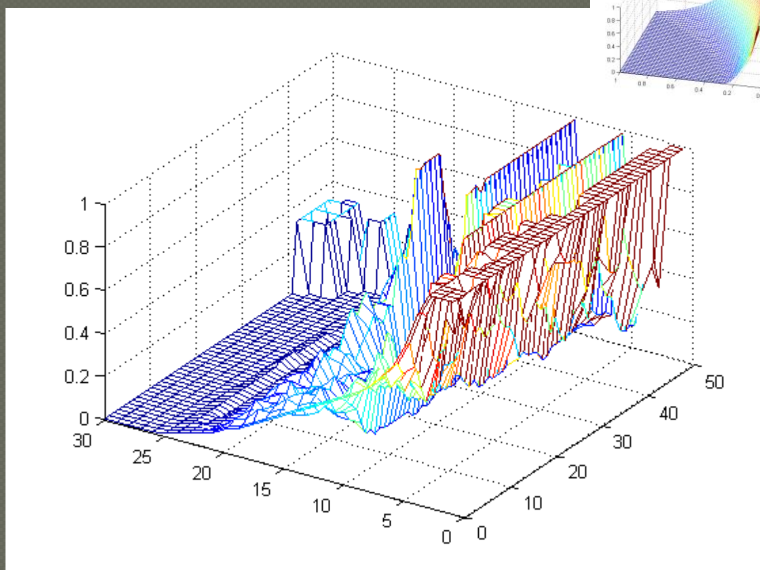
The stochastic process is started in 30 different initial points (30 samples), from 1000 BGN's to 30000 BGN's. In the figures are shown the processes with optimal stochastic control and without optimal control:

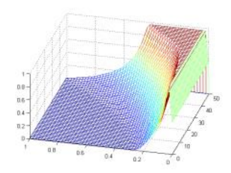


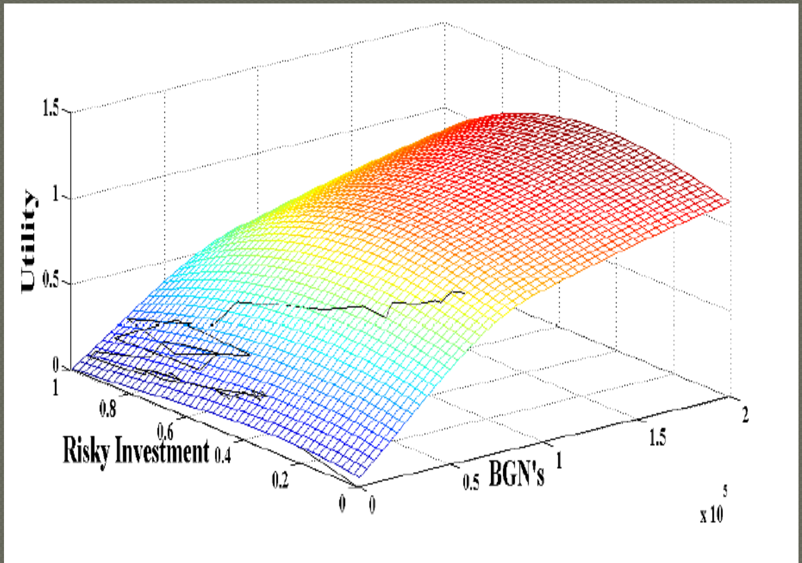


Continuous-time financial market –INVESTMENT BUSINESS STRATEGY

The figure presents 30 samples of optimal control solution and deviations of the control value. The figure shows not only the optimal control investment but possibilities for deduction of an investment business strategy in agreement with the decision maker’s preferences.



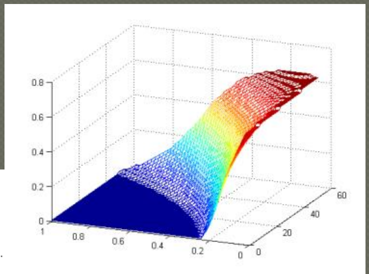


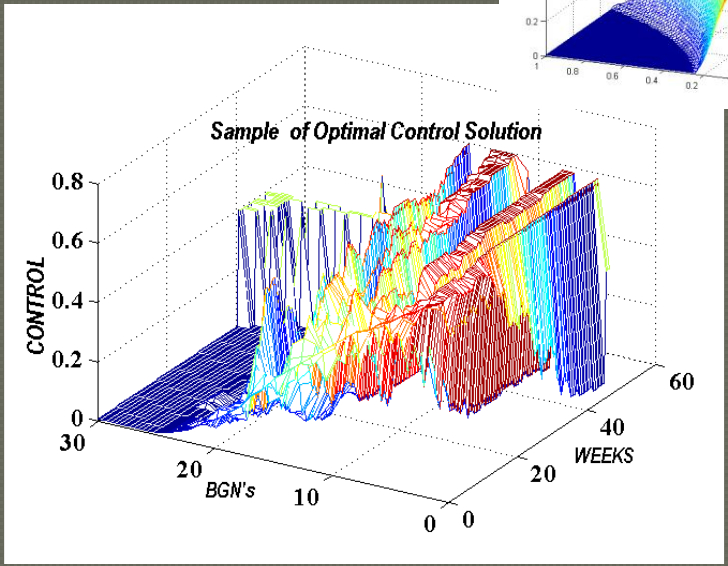


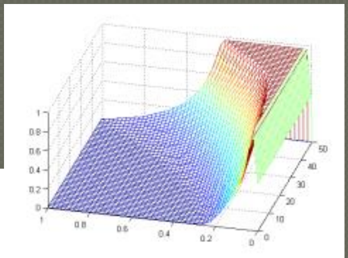
From the exposed follows that if the initial amount is between 1000 BGN’s and 10000 BGN’s is recommended to invest the entire initial wealth in the risky investment. If the initial wealth is between 25000 BGN’s and 30000 BGN’s it is advisable to start with the non-risky investment and after the 30 or 40 weeks to invest a part of the wealth in the risky-investment.

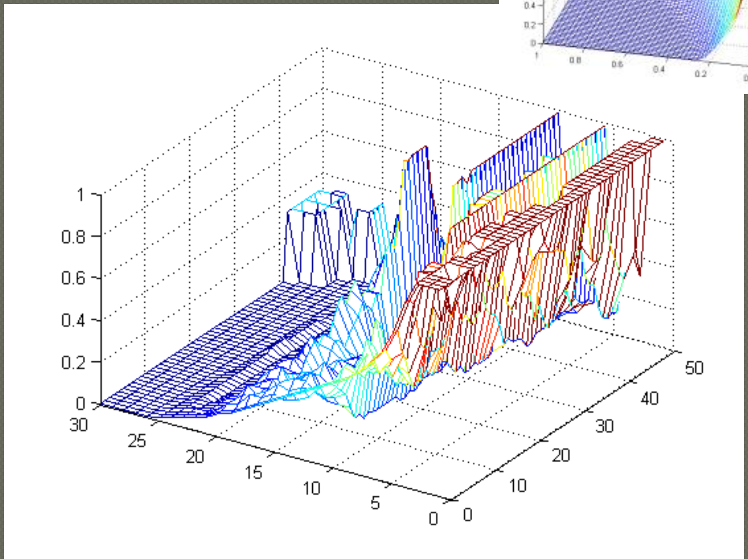
Continuous-time financial market –INVESTMENT BUSINESS STRATEGY

In the slide are shown comparisons between the two attributtes control solution and the clasical control solution.









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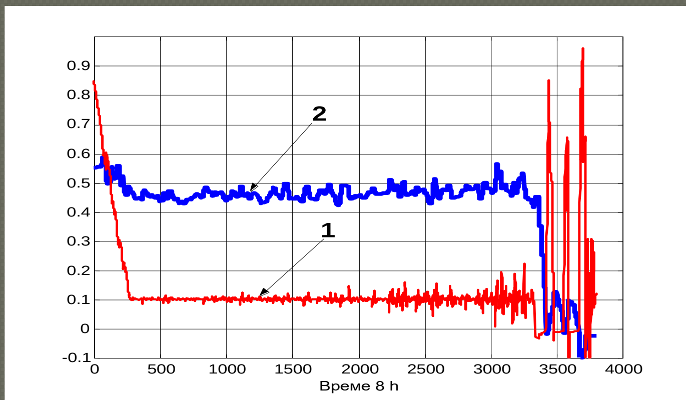
OPTIMAL CONTROL OF A FED-BATCH FERMENTATION PROCESS (PONTRYAGIN MAXIMUM PRINCIPLE, FILTRATION AND STOCHASTIC CONTROL: MONOD-WANG MODEL)



Data prezentation (от модел)

1. Substrate concentration S;
2. Specific growth rate μ;

$$\dot{X} = \mu X - \frac{F}{V_0} X$$

$$\dot{S} = -\frac{1}{y}\mu X + (S_0 - S)\frac{F}{V_0}$$

$$\dot{\mu} = m\left(\mu_m \frac{S}{K_s + S} - \mu\right) + v$$

$$\dot{V}_0 = F$$

MODEL (fed-batch - полупериодичен):

X -biomass concentration,

S -substrate concentration,

μ -specific growth rate,

*K*s - Mihaelis-Menten constant,

v -white noise ,

S_o substrate concentration in the feed,

m - coefficient,

F ”is the substrate-feed rate”, input ,

μ_m (*T*, *pH*) maximal value of the specific growth rate (as function of temperature *T* and the acidity *pH*),

y -coefficient.

EQUIVALENT MODELS (DIFFEOMORPHIC TRANSFORMATIONS): CONTINUOUS PROCESS

With the use of the GS algorithm the non-linear Wang-Monod model is presented in Brunovsky normal form (нормална форма на Бруновски):

$$\dot{X} = \mu X - DX$$

$$\dot{S} = -\frac{1}{y}\mu X + (S_0 - S)D$$

$$\dot{\mu} = m(\mu_m \frac{S}{K_s + S} - \mu) + v$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \Phi(x_1, x_2, x_3) = \begin{pmatrix} \frac{x_1}{S_0 - x_2} \\ x_2 \\ x_3 \end{pmatrix}$$

Трансформция 2

Diffeomorphic transformations:

$$Y_1 = u_1$$

$$Y_2 = u_3(u_1 - ku_1^2)$$

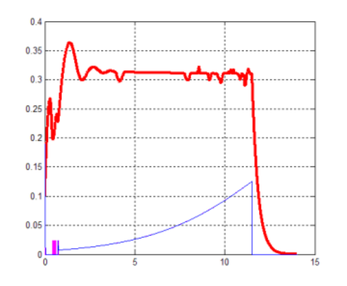
$$Y_3 = u_3^2(u_1 - 3ku_1^2 + 2k^2u_1^3) + +m(\mu_m \frac{u_2}{(K_S + u_2)} - u_3)(u_1 - ku_1^2)$$

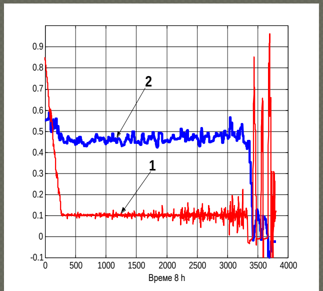
$$\dot{Y}_1 = Y_2$$

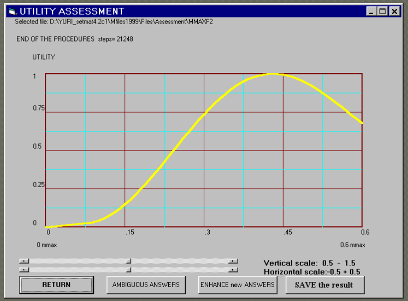
$$\dot{Y}_2 = Y_3$$

$$\dot{Y}_3 = W$$

PONTRYAGIN MAXIMUM PRINCIPLE, FILTRATION AND CONTROL FED-BATCH FERMENTATION PROCESS AND STOCHASTIC CONTROL: Monod-Wang model

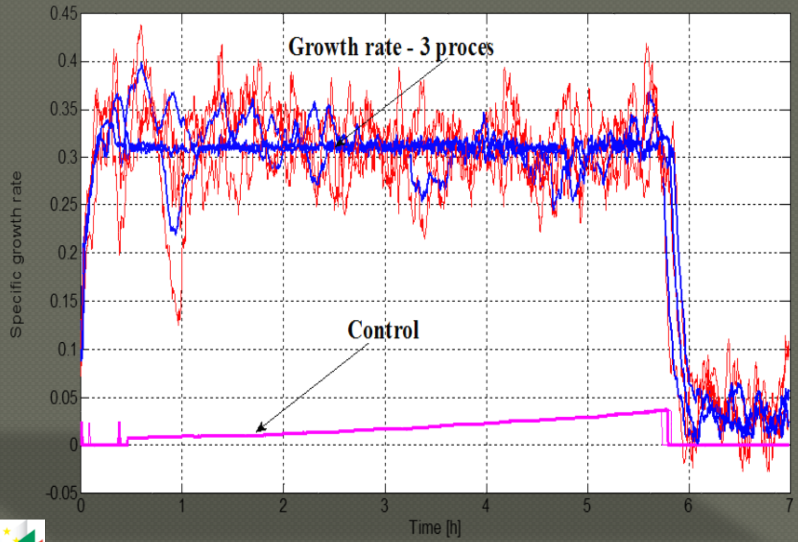


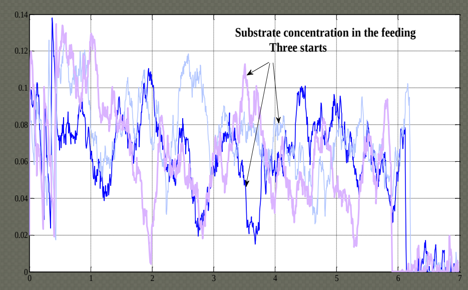


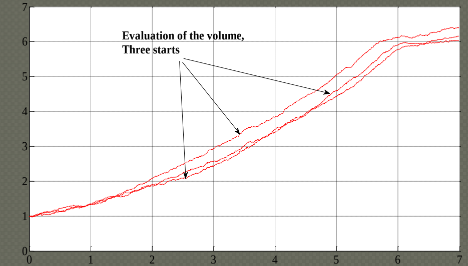


Utility function- Growt rate versus Utility

- The Calman filtration permits determination of the vector space as conditional expectations w.r.t at (x; t);
- The optimal control is the same as in the deterninistic case.





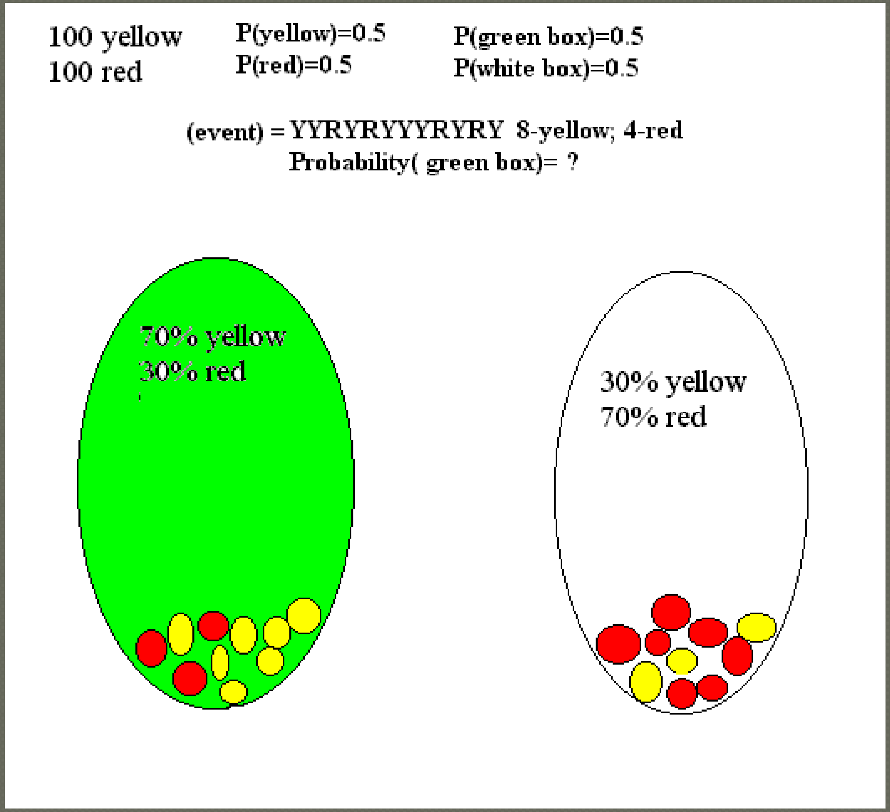


Assessment of human factors in complex systems: mathematical aspects of the human judgment

SUBJECTIVE PREFERENCES, VALUES AND DECISIONS: STOCHASTIC APPROXIMATION APPROACH

- *The expected utility theory is one of the approaches for assessment and utilization of qualitative, conceptual information. The expected utility approach allows for the expert preferences being taken account of in complex systems and problems. The expert values are not directly oriented to the problem under consideration and are expressed with probability and subjective uncertainty. Due to that specific algorithms have to be developed. Possible way to the problem are recurrent stochastic algorithms for evaluation of expert utilities and value functions or for evaluation of subjective probabilities. This mathematical approach give possibilities for development of value-driven decision support systems.*
- ***Key words:*** *Expected utility, value, subjective probabilities, preferences, stochastic approximation*

Subjective probability (prof. Raiffa's example)

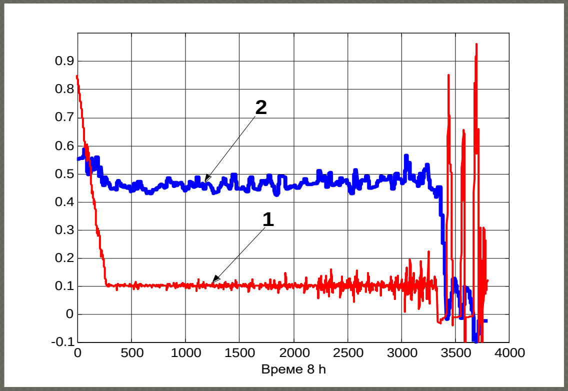


- Subjective uncertainty
- P(green box/YYRYRYYYRYRY)=?

Decision making and Subjective probability

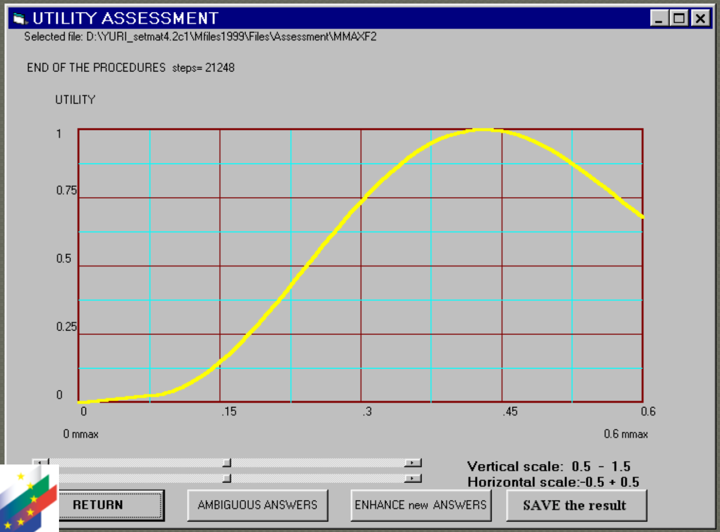
- P(green box/YYRYRYYYRYRY)=(P(event/green box).P(green box))/ (P(event/green box).P(green box) + +P(event/white box).P(white box))=
  =((0,7)^8.(0,3)^4.0,5) / ((0,7)^8.(0,3)^4.0,5 +(0,7)^4.(0,3)^8.0,5) =
  = 0.964

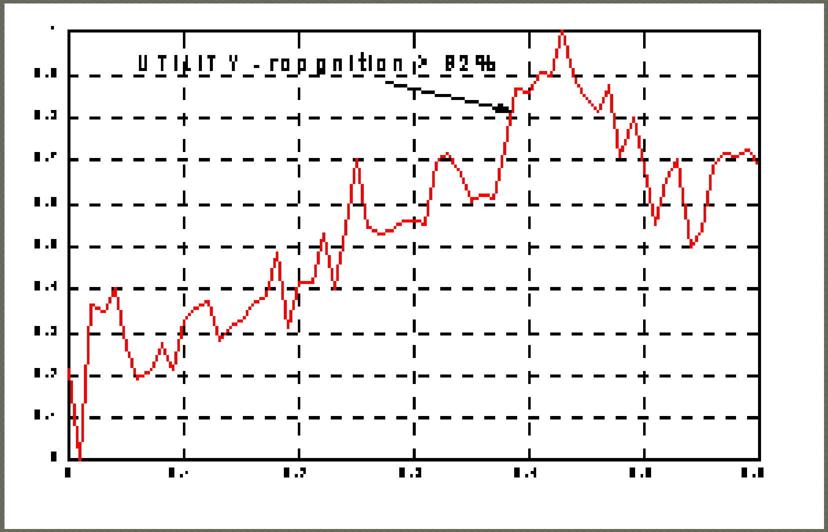
ПОСТРОЯВАНЕ НА ЕКСПЕРТНАТА ПОЛЕЗНОСТ- Assessment of the human factor



- Modelling: 1.Substrate concentration S;
  2.Specific growth rate μ;
- Expert utility evaluation - $U(\mu)$:

$$U(\mu) = \sum_{i=0}^{6} c_i \mu^i \ .$$





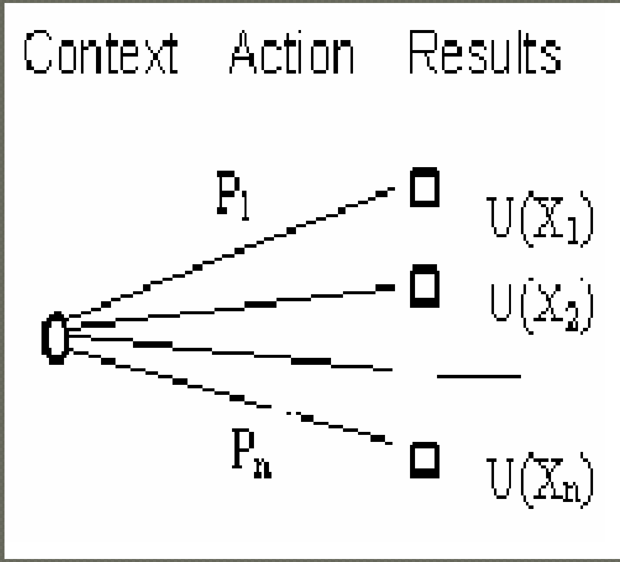
EXPECTED UTILITY EVALUATION

Let **X** is the set of final solutions - alternatives and **P** is a subset of discrete probability distributions over **X**. A utility function is any function u(.) for which is fulfilled:

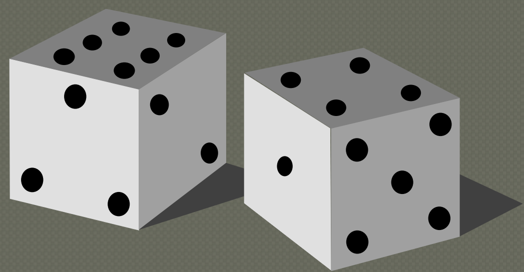
$$(p \succ q,\ (p,q) \in \mathbf{P}^2) \Leftrightarrow \left(\int u(.)dp > \int u(.)dq\right),\ p \in \mathbf{P},\ q \in \mathbf{P}.$$

According von Neumann and Morgenstern the formula above means that the mathematical expectation of u (.) is the quantitative measure concerning the expert’s preferences for the probability distributions **P** over **X**. Through ($\succ$) the expert’s preferences over **P** are expressed, including those over **X**: $\mathbf{X} \subseteq \mathbf{P}$. The "indifference" relation (~) is defined as: $x \sim y \Leftrightarrow \neg(\ (x \succ y) \vee (x \prec y)\ )$.

It is well known that the existence of the utility function u(.) over **X** determines the "preference" relation ($\succ$) as a negatively transitive and asymmetric one:

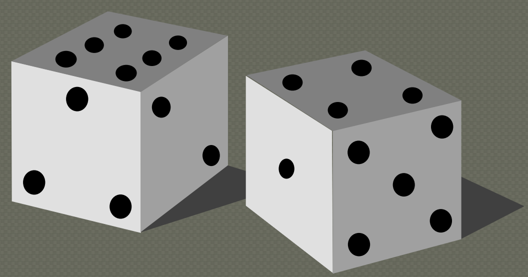


UTILITY ASSESSMENT - PROCEDURE



$$\begin{array}{ccccc} \alpha & (1-\alpha) & & & 1 \\ \updownarrow & \updownarrow & & & \updownarrow \\ < \mathbf{x}, & \mathbf{y}, & \alpha > & (\succ \text{ or } \prec \text{ or } \sim) & \mathbf{z} \end{array}$$

UTILITY ASSESSMENT - PROCEDURE



The expert compares the "lottery" $<x,y,\alpha>$ with the alternative $z\in\mathbf{X}$ ("better-$\succ$", "worse - $\prec$" or "can't answer or equivalent - $\sim$"):

$$<x,y,\alpha> \quad (\succ \text{ or } \prec \text{ or } \sim) \quad z.$$

The expert compares the "lottery" $<x,y,\alpha>$ with z (the "learning point" (x,y,z,α)) and with the probability $D_1(x,y,z,\alpha)$ relates it to the set

$\mathbf{A_u} = \{(x,y,z,\alpha)/\ (\alpha u(x)+(1-\alpha)u(y))>u(z)\}$

or with the probability $D_2(x,y,z,\alpha)$ - to the set

$\mathbf{B_u} = \{(x,y,z,\alpha)/\ (\alpha u(x)+(1-\alpha)u(y))<u(z)\}$.

At each "learning point" (x,y,z,α) a juxtaposition can be made: $f(x,y,z,\alpha)=1$ for ($\succ$), $f(x,y,z,\alpha)=-1$ for ($\prec$) and $f(x,y,z,\alpha)=0$ for ($\sim$) (subjective characteristic of the expert which contain the uncertainty of expressing his/her preferences). Let the appearance of the "learning" sequence $((x,y,z,\alpha)_1, (x,y,z,\alpha)_2,\ldots,(x,y,z,\alpha)_n,..)$ has the probability distribution $F(x,y,z,\alpha)$. Then the probabilities $D_1(x,y,z,\alpha)$ and $D_2(x,y,z,\alpha)$ are the mathematical expectation of f(.) over $\mathbf{A_u}$ and $\mathbf{B_u}$.

We approximate $D'(x,y,z,\alpha)$ by the function $G(x,y,z,\alpha)=(\alpha g(x)+(1-\alpha)g(y)-g(z))$, where. the function $G(x,y,z,\alpha)$ is positive over $\mathbf{A_u}$ and negative over $\mathbf{B_u}$ depending on the degree of approximation of $D'(x,y,z,\alpha)$. ***In such case g(x) is an approximation of the empirical expert utility u*(.).***

APPROXIMATION OF THE EMPIRICAL EXPERT UTILITY U*(.)

The following is fulfilled for f(.): $f=D'+\xi$, , $M(\xi/x,y,z,\alpha)=0$, $M(\xi^2/x,y,z,\alpha)<d$, $d\in\mathbf{R}$ It is assumed that the utility u(.) fulfils the condition:$\int u^2(x)dF_x < +\infty$, where F_x is the conditional distribution over **X** related with $F(x,y,z,\alpha)$. The utility u(.) fulfils:

$$u(x) \underset{L_2}{=} \sum_i r_i \Phi_i(x) \qquad g(x) = \sum_i c_i \Phi_i(x)$$

$r_i\in\mathbf{R}$**,** where $(\Phi_i(x))$ is a row of chosen polynomials.

APPROXIMATION ALGORITHM:

$$c_i^{n+1} = c_i^n + \gamma_n \left[D'(t^{n+1}) - \overline{(c^n, \Psi(t^{n+1}))} + \xi^{n+1} \right] \Psi_i(t^{n+1})$$

$$\sum_n \gamma_n = +\infty, \ \sum_n \gamma_n^2 < +\infty, \ \forall n, \ \gamma_n \geq 0$$

$$c_i^{n+1} = c_i^n + \gamma_n \left[f(t^{n+1}) - \overline{(c^n, \Psi(t^{n+1}))} \right] \Psi_i(t^{n+1})$$

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$r_i \in$ **R,** where $(\Phi_i(x))$ is a row of chosen polynomials.

APPROXIMATION ALGORITHM:

$$c_i^{n+1} = c_i^{\,n} + \gamma_n \left[D'(t^{n+1}) - \overline{(c^n, \Psi(t^{n+1}))} + \xi^{n+1} \right] \Psi_i(t^{n+1})$$

$$\sum_n \gamma_n = +\infty, \sum_n \gamma_n^2 < +\infty, \forall n, \gamma_n \geq 0$$

$$c_i^{n+1} = c_i^{\,n} + \gamma_n \left[f(t^{n+1}) - \overline{(c^n, \Psi(t^{n+1}))} \right] \Psi_i(t^{n+1})$$

CONVERGENCE OF THE STOCHASTIC PROCEDURES

$$c_i^{n+1} = c_i^n + \gamma_n \left[D'(t^{n+1}) - \overline{(c^n, \Psi(t^{n+1}))} + \xi^{n+1} \right] \Psi_i(t^{n+1})$$

$$\sum_n \gamma_n = +\infty, \ \sum_n \gamma_n^{\mathbf{2}} < +\infty, \ \forall n, \gamma_n \geq 0$$

$$c_i^{n+1} = c_i^n + \gamma_n \left[f(t^{n+1}) - \overline{(c^n, \Psi(t^{n+1}))} \right] \Psi_i(t^{n+1})$$

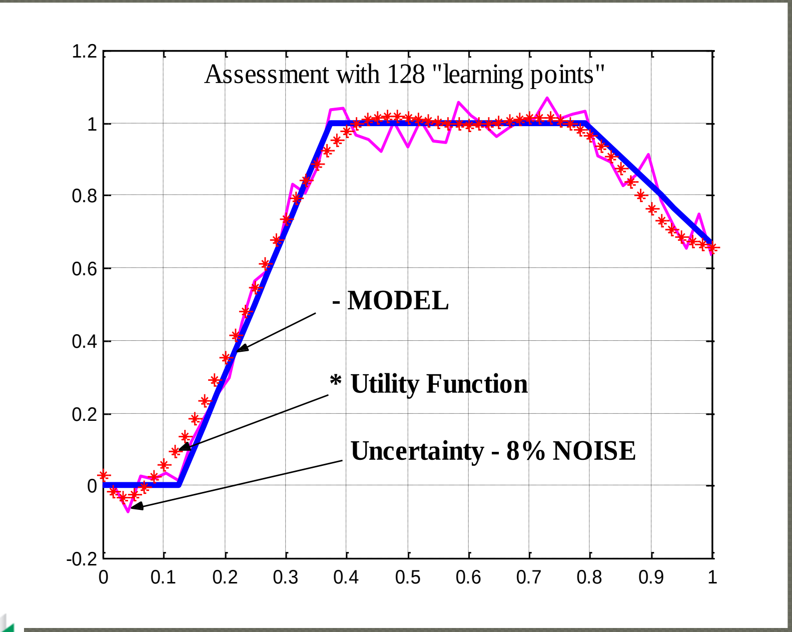
- **THEOREM1: Let in the recurrent algorithm $(t^1,\dots,t^n,\dots)$ is a sequence of independent random vectors $t^n=(x,y,z,\alpha)$, with distribution F and the sequence of random values $\xi^1,\xi^2,..,\xi^n,..$ satisfies the conditions: $M(\xi^n/(x,y,z,\alpha),c^{n-1})=0$, $M((\xi^n)^2/(x,y,z,\alpha),c^{n-1})<d$, $d\in R$. Let $\Psi(t)$ is limited by a constant independent from t $\|\Psi(t)\|<$ cte. From the recurrent procedure follows:**

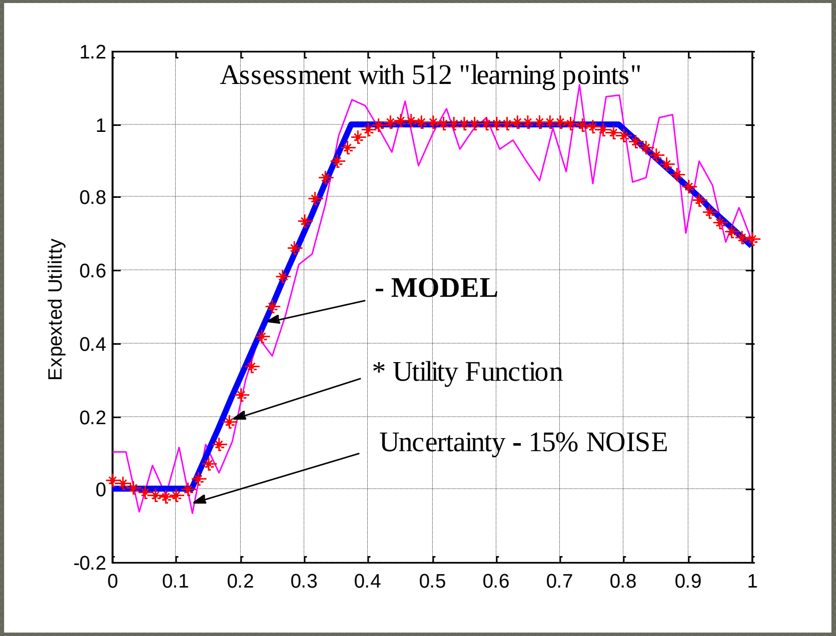
$$J_{D'}(G^n(x,y,z,a)) = M\left(\int_{D'(t)}^{G^n(t)} (\overline{y} - D'(t))dy\right) = \int \left(\int_{D'(t)}^{G^n(t)} (\overline{y} - D'(t))dy \right) dF \xrightarrow[n]{p.p.}$$

$$\xrightarrow[n]{p.p} \inf_{s(t)} \int \left(\int_{D'(t)}^{S(t)} (\overline{y} - D'(t)dy \right) dF$$

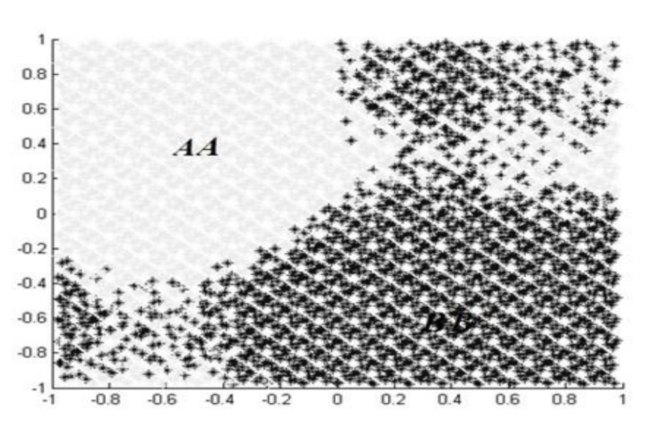
PROGRAM REALIZATION AND NUMERICAL EXPERIMETS

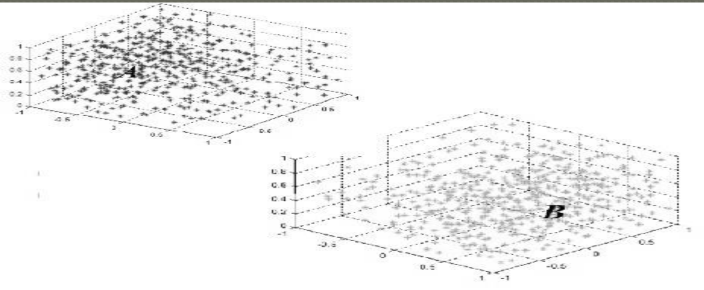
- The dialogue between the expert and the computer was modeled, the expert being replaced by a function (the blue line). The violet seesaw line models the uncertainty in the expert answers. This function determines the answers about the "learning points" - $\mathbf{A_u}$ or $\mathbf{B_u}$. The approximations are shown in the figures.
- The learning points (x,y,z,α) are set with a pseudo random Lp_τ Sobol's sequence

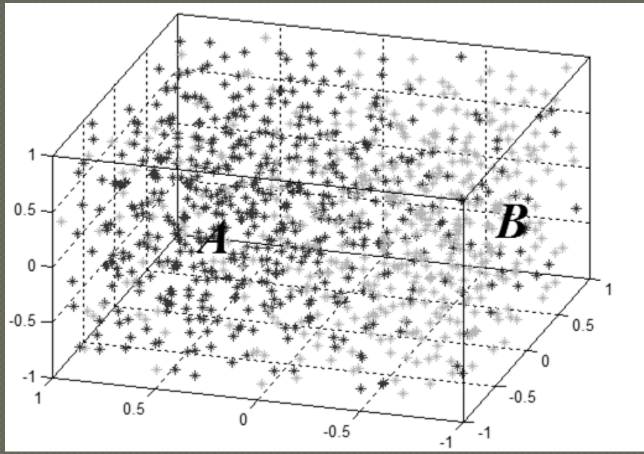


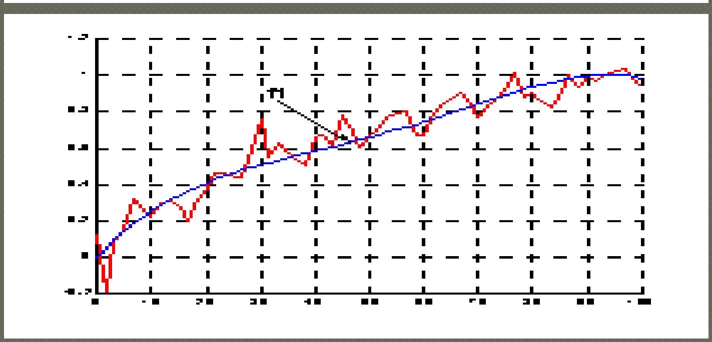


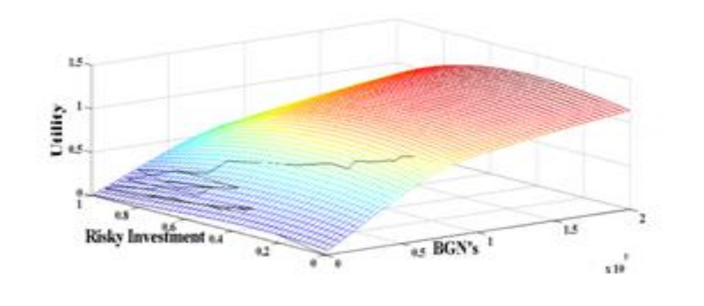
Preferences, Utility, Gambling approach and Stochastic Evaluation











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Decision sciences - DEFINITIONS

- **Harvard University Ph.D. Program in Health Policy**

- **DECISION SCIENCES CONCENTRATION**

  – ***Decision sciences are the collection of quantitative techniques that are used for decision-making at the individual and collective level. They include decision analysis, risk analysis, cost-benefit and cost-effectiveness analysis, decision modeling, and behavioral decision theory, as well as parts of operations research, microeconomics, statistical inference, management control, cognitive and social psychology, and computer science. The concentration in decision sciences prepares students for research careers that involve the application of these methods to health problems. Examples of research topics in health decision sciences include: cost-effectiveness analysis of medical technologies; optimal screening policies for cancer and other chronic diseases; risk-benefit analysis of advanced airbag designs; benefit-cost analysis of environmental regulations; measurement and evaluation of health outcomes, including quality of life; comparative risk analysis of alternative fuels for motor vehicles; policy simulation modeling of diseases such as AIDS, tuberculosis, cancer, and asthma; and optimal resource allocation for biomedical research.***

http://cepa.newschool.edu/het/essays/uncert/uncertcont.htm

- **HISTORY and CONTRIBUTORS: Expected Utility Hypothesis**
  - John von Neumann, 1903-1957;Oskar Morgenstern, 1902-1976; Jacob Marschak, 1898-1977; Harry M. Markowitz, 1923;
- **Subjective Expected Utility Theory**
  - Frank P. Ramsey, 1903-1930; Bruno de Finetti, 1906-1985. - "La Prйvision: ses lois logiques, ses sources subjectives", 1937, *Annales de l'Institut Henri Poincarй*
    - "Le vrai et le probable", 1949, *Dialectica*
    - "Sull' impostazione assiomatica del calcolo delle probabilita", 1949, *Annali Triestini*
    - "Recent Suggestions for the Reconciliations of the Theories of Probability", 1951, in Neyman, editor, *Proceedings of Second Berkeley Symposium*
    - "Sulla Preferibilita", 1952, *Giornale degli Economisti*.
    - *Theory of Probability*, 1974-5.
    - Leonard J. Savage, 1917-1971 ;
    - Robert J. Aumann, 1930-;…………

http://cepa.newschool.edu/het/essays/uncert/uncertcont.htm

HISTORY and CONTRIBUTORS : Non-Expected Utility - Theory

- Maurice Allais, 1911;
- Peter C. Fishburn;
- Mark J. Machina;
- Edi Karni ;
- K.R. MacCrimmon;
- Daniel Kahnemann: --- http://www.almaz.com/nobel/; (2002)
- David Tversky and Experimental Economics;
- Ralph Kenneey;
- ……………………………………….

NOBEL PRIZES: *decision science* and *utility theory*

- ***NOBEL PRIZES***:
  - ***Arrow K.*** award ***1972.*** *Arrow* page at Britannica Guide to the Nobel Prizes;
  - ***Debreu G.*** award ***1983.*** "Least Concave Utility Functions", 1976, JMathE. Guide to the Nobel Prizes.
  - ***Allais M.*** award ***1988***. Nobel prize went to French economist ***Maurice Allais.***
  - ***Kahneman D.*** award ***2002***. Research in decision making: ***psychologist Daniel Kahneman.***

- ***Recent Nobel Prizes Highlight Decision Sciences:*** **Decision sciences as a discipline has received considerable attention in recent years due to a number of Nobel prizes in economics. The seminal research by these Nobel laureates has inspired much of the current research in decision sciences. For example, the *1988 Nobel prize went to French economist Maurice Allais*, who showed that individuals (even economists!) make some choices that are inconsistent with the "axioms of rationality" that economic models assume. Reinhard Selten, John Harsanyi, and John Nash shared the prize in 1994 for their foundational contributions to game theory, a normative approach to analyzing competitive decisions. In *2002 the* award went to two scientists who pioneered the descriptive side of research in decision making: *psychologist Daniel Kahneman* for bringing insights from psychological research into the study of decision making, and Vernon Smith for developing laboratory methods to study decision making in market settings.**

Прототип на информационна система

- **Обхват на техническото задание**
- Обхватът включва:
- Таблици, графично и структорно представяне на процедури и мтоди за анализ и извличане на данни като кардинална експертна информация (предпочитания изразени в “machine learning”диалог) за целите на осигуряване на алгоритмична и програмната среда;
- Предоставяне на следните съпътстващи услуги:
  - Демонстриране работата на предходни прототипи на подобни системи разработени в БАН;
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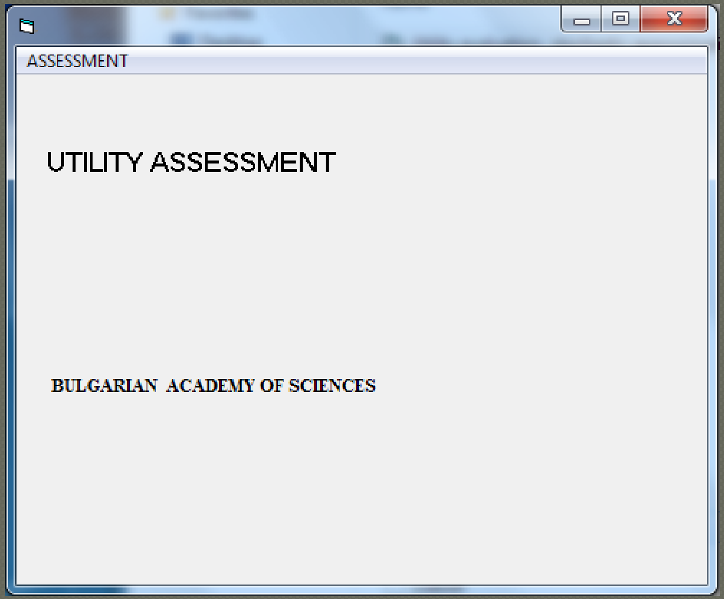

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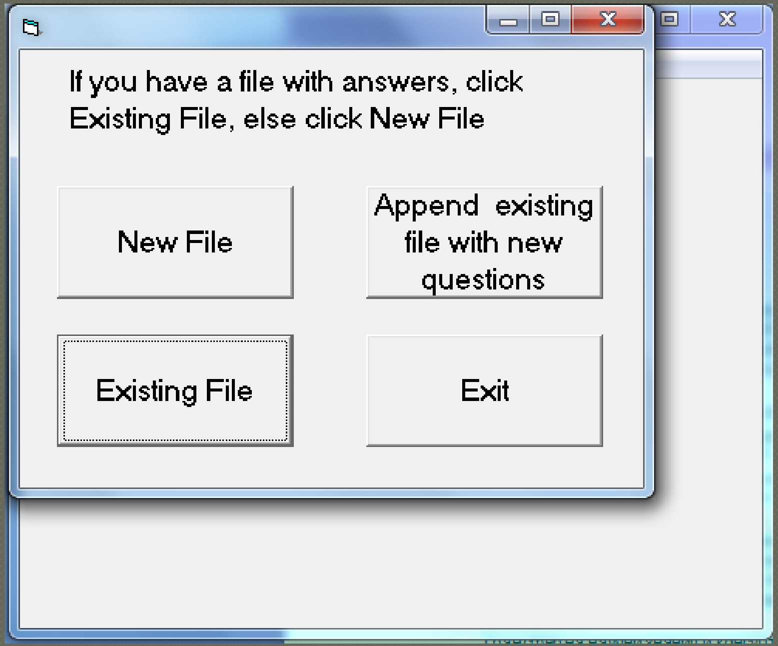

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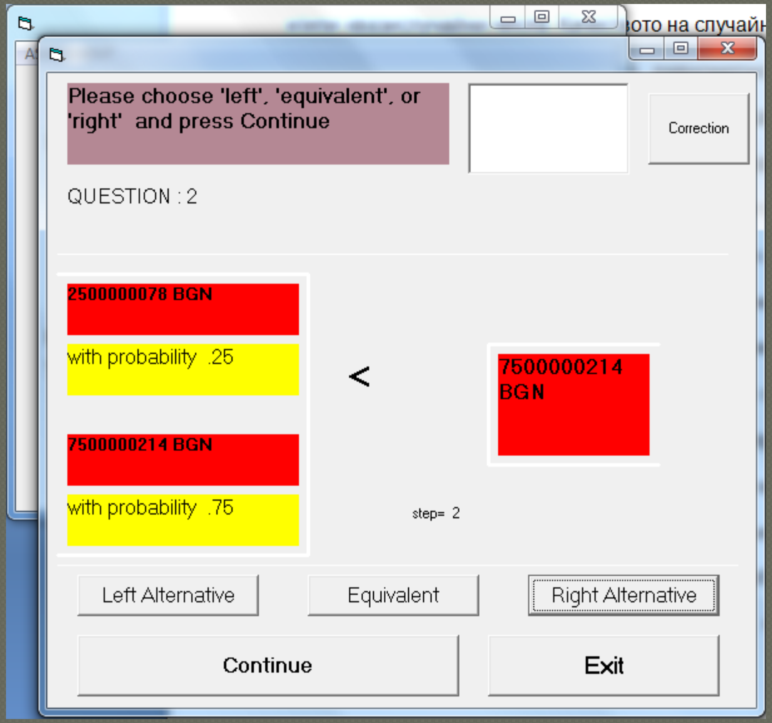

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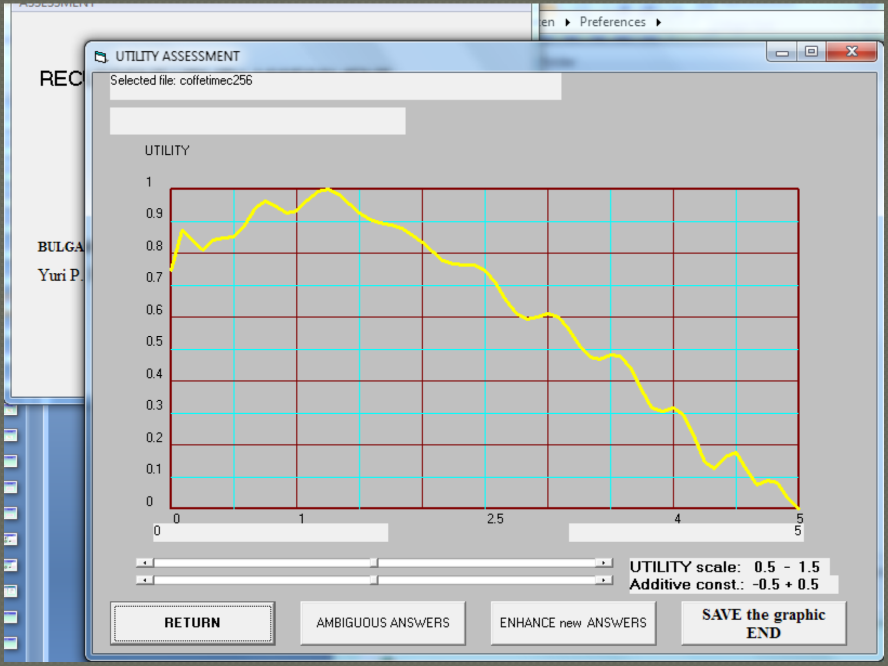

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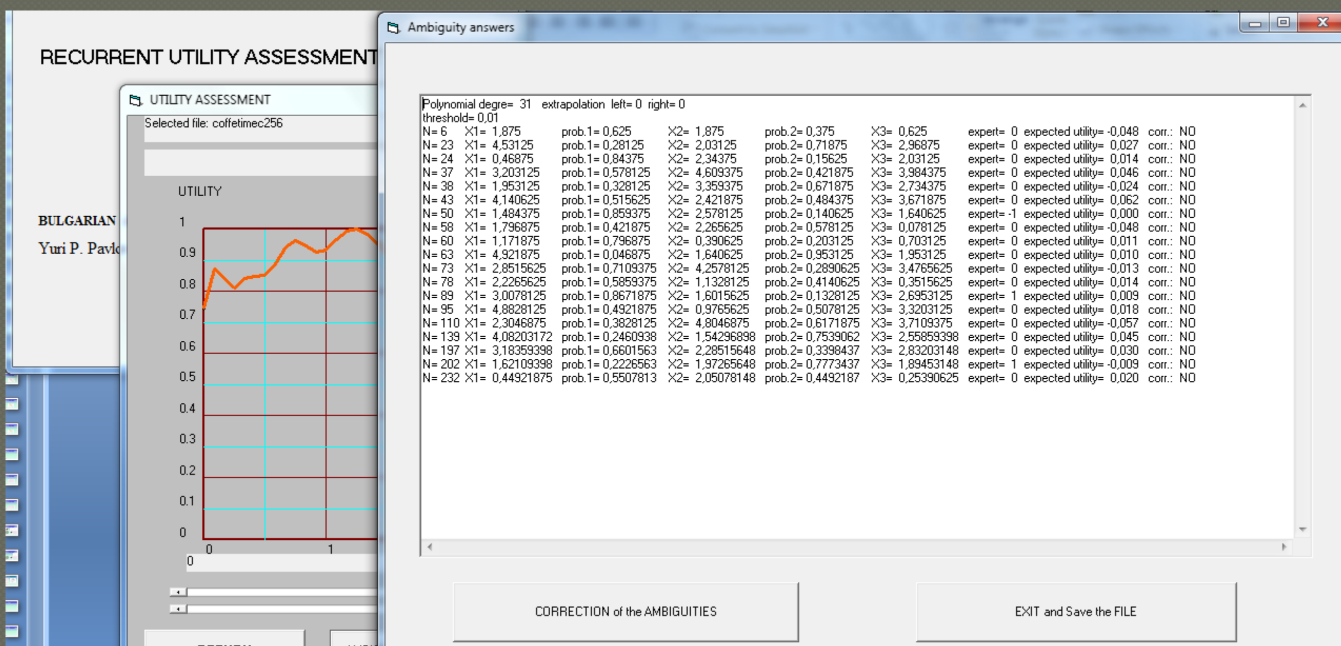

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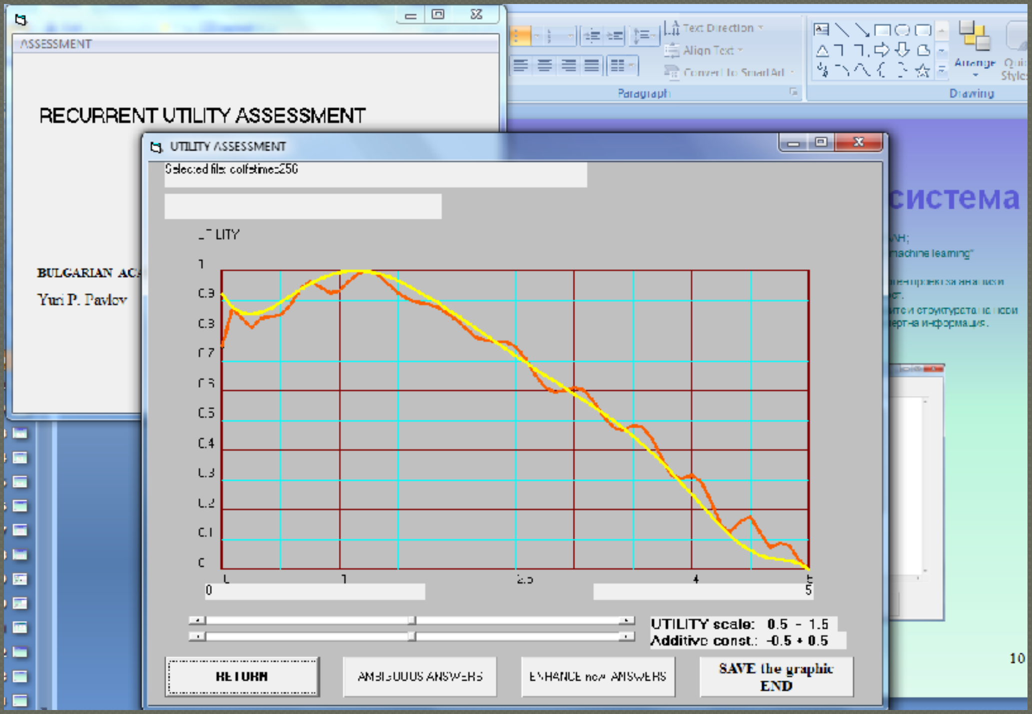

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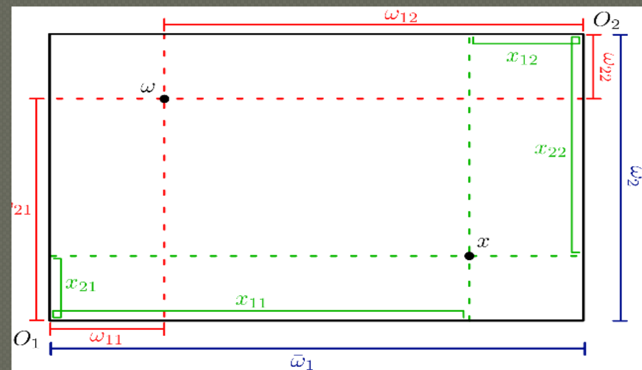
GENERAL COMPETITIVE EQUILIBRIUM

Edgeworth box and expert utility (concemer's indifference curves)



The two consumers are each endowed (born with) a certain quantity of goods. They have locally non-satiated preferences and initial endowments: $(w_1, w_2) = ((w_{11}, w_{21}), (w_{12}, w_{22}))$.

In the box the vector $w = (\bar{w}_1, \bar{w}_2)$ is the total quantities of the two goods:

$\bar{w}_1 = w_{11} + w_{12}, \bar{w}_2 = w_{21} + w_{22}$

An allocation $x=(x_1, x_2) = ((x_{11}, x_{21}), (x_{12}, x_{22}))$ represents the amounts of each good that are allocated to each consumer.

$\bar{w}_1 = x_{11} + x_{12}, \bar{w}_2 = x_{21} + x_{22}$

A no wasteful allocation $x=(x_1, x_2)$ is one for which is fulfilled:

In terms of aggregate amounts of the two agents, the total amounts needs to be equal to the total endowment of the two goods. Every point in the box represents a complete allocation of the two goods to the two consumers

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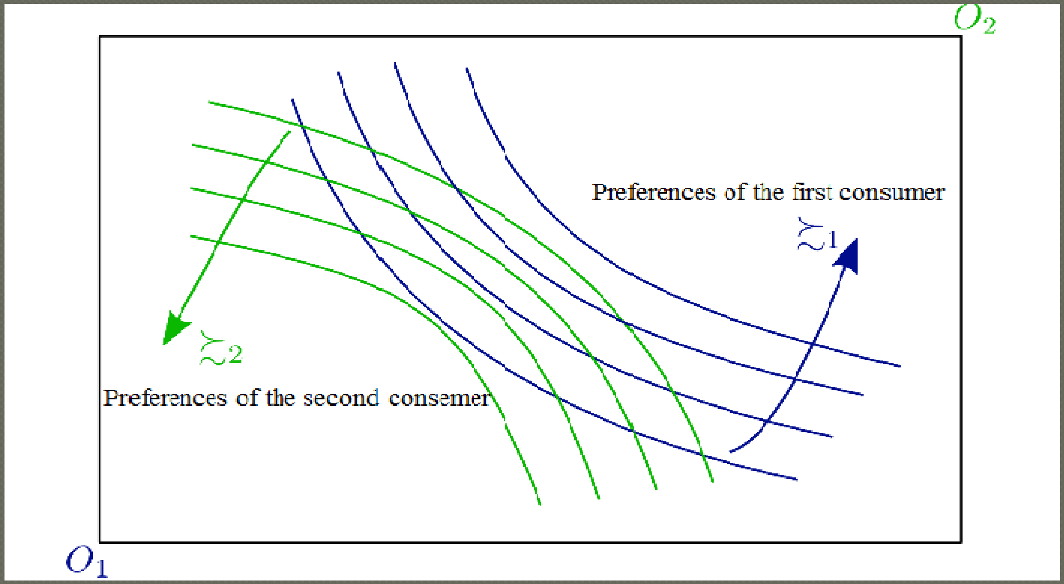
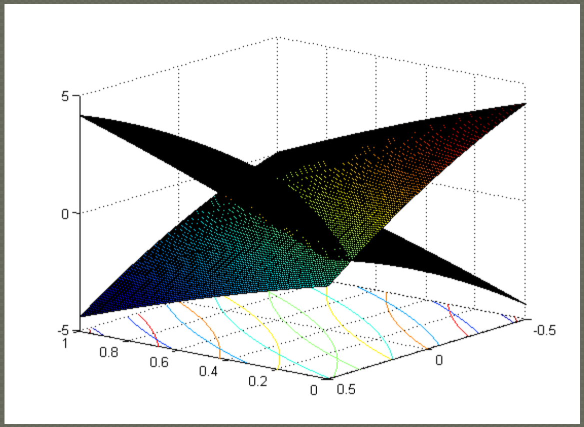
GENERAL COMPETITIVE EQUILIBRIUM
Edgeworth box

Each of the two individuals maximizes his utility according to his preferences (Raiffa, 1968; Keeney, 1993, Ekeland, 1983). The demand functions or the utility functions which represent consumers' preferences are convex and continuous, because in accordance with the equilibrium theory the preferences in are continuous, monotone and convex as is shown in the figure (Ekeland, 1983).





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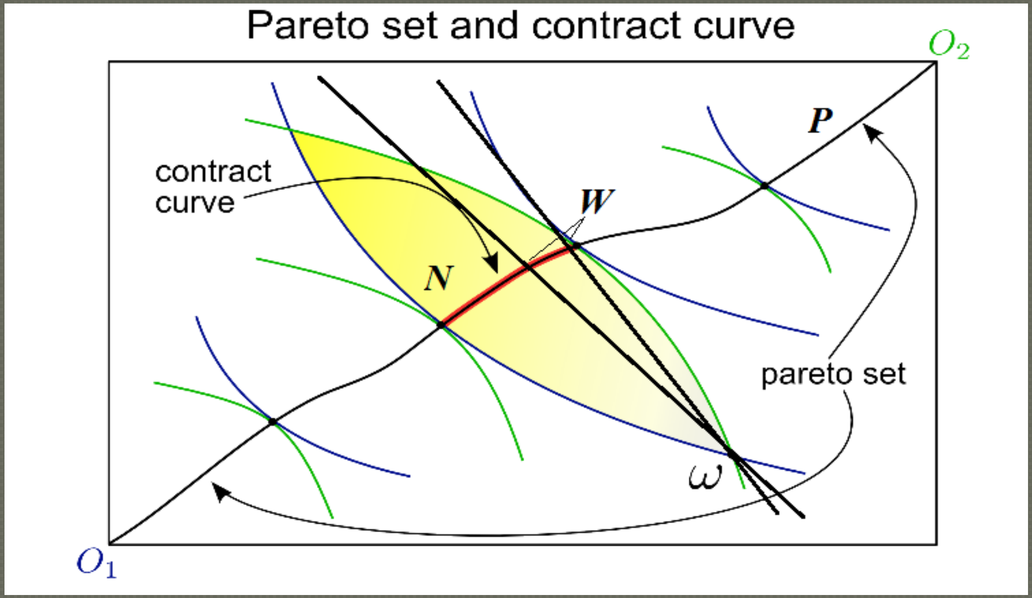
GENERAL COMPETITIVE EQUILIBRIUM

Edgeworth box

An allocation is said to be Pareto efficient, or Pareto optimal, if there is no other feasible allocation in the Edgworth economy for which both are at least as well off and one is strictly better off. The locus of points that are Pareto optimal given preferences and endowments is the Pareto set, noted as ***P*** in the figure. The part of the Pareto set in which both consumers do at least as well as their initial endowments is the Contract curve shown in the figure and noted as ***N*** (kernel of the market game).

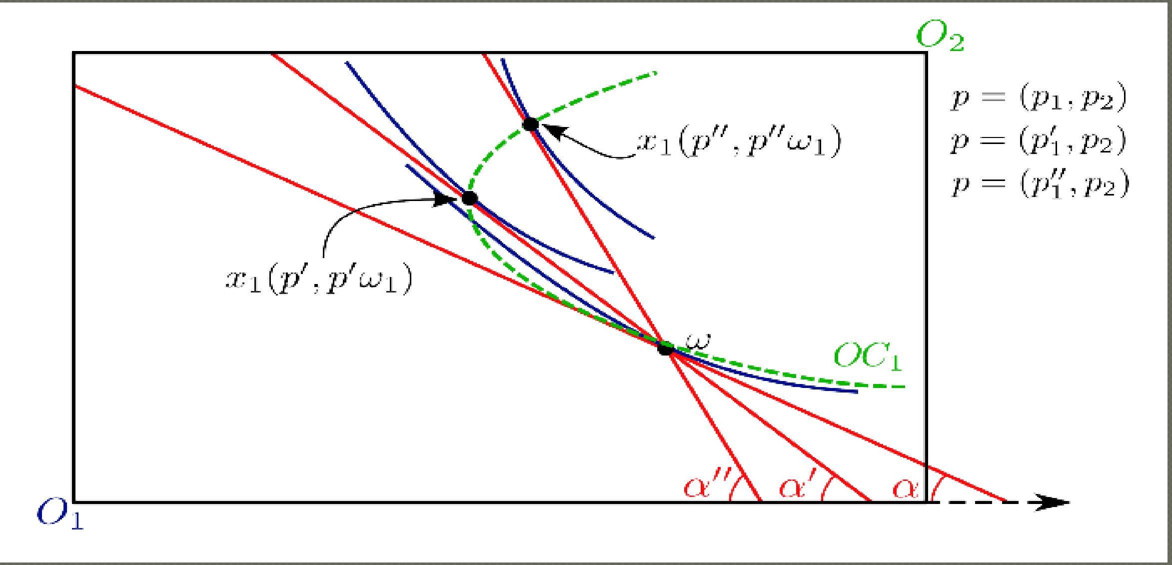


GENERAL COMPETITIVE EQUILIBRIUM
Edgeworth box

The consumers take prices of the two goods $p = (p_1, p_2)$ as given and maximize their utilities. The budget (income) set $B_i(p)$ of each consumer is given by:

$$B_i(\mathbf{p}) = \{\mathbf{x_i} \in \mathbf{R}^2_+ \ / \ \mathbf{px_i} \le \mathbf{pw_i}\}, \ \mathbf{i} = 1, 2.$$

For every level of prices, consumers will face a different budget set. The locus of preferred allocations for every level of prices is the consumer's offer curve.

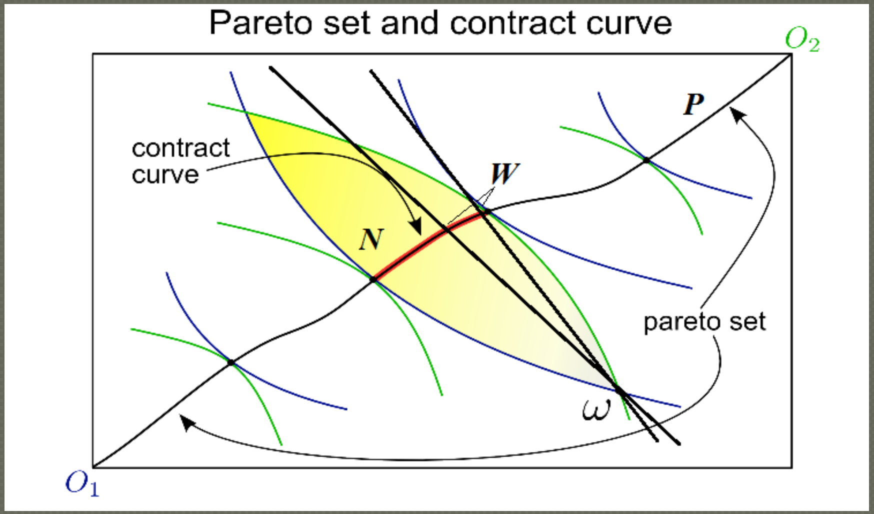


GENERAL COMPETITIVE EQUILIBRIUM

Edgeworth box

We are interested in the equilibrium point(s) of the process of exchange where is fulfilled the Walrasian equilibrium. Walrasian equilibrium is a price vector p and an allocation x such that, for every consumer the prices (i.e. the terms of trade) are such that what one consumer (group of consumers) wants to buy is exactly equal to what the other consumer (group of consumers) wants to sell. In other words, consumers' demands are compatible with each other. We note the locus of points that are in Walrasian equilibrium as ***W***. In still other words, the quantity each consumer wants to buy at the given market prices is equal to what is available on the market. The following inclusion is true in the Edgworth economy: $\boldsymbol{P} \supset \boldsymbol{N} \supset \boldsymbol{W}$.

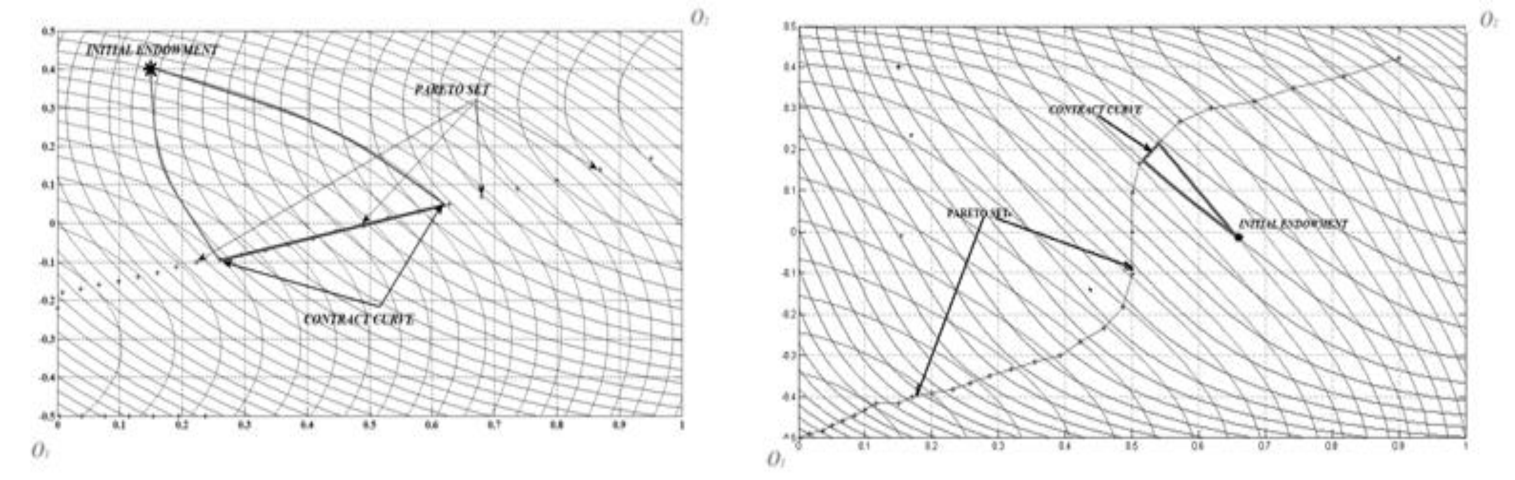
In that sense a contract curve in the Edgeworth Box shows an exchange market in equilibrium and this is a particular representation of the Walrasian equilibrium theorem.



GENERAL COMPETITIVE EQUILIBRIUM
Edgeworth box

In that way we can state and solve the market-clearing equilibrium in principle and we can determine the contract curve and the Walrasian set in the Edgeworth economic. The set of the Walrasian equilibriums ***W*** and the appropriate prices $p = (p_1, p_2)$ are calculated based on the determined demand utility functions and this is a meaningful prognosis of the market equilibrium.

In that way can be forecast, based on the preferences, the competitive market equilibrium allocations $x=(x_1, x_2)$, $(x_1, x_2) = ((x_{11}, x_{21}), (x_{12}, x_{22}))$ and the appropriate prices $p = (p_1, p_2)$. The contract curves are specified on the individual consumers' preferences and show that there are possibilities to be made mutually advantageous trades. This means that one could unilaterally negotiate a better arrangement for everyone.



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Благодаря за вниманието!

Thank you for your attention!